JOURNAL

OF THE

ASIATIC SOCIETY OF BENGAL.

Part II.—PHYSICAL SCIENCE.

No. I.—1876.

I —*On the General Theory of Duplex Telegraphy.*—

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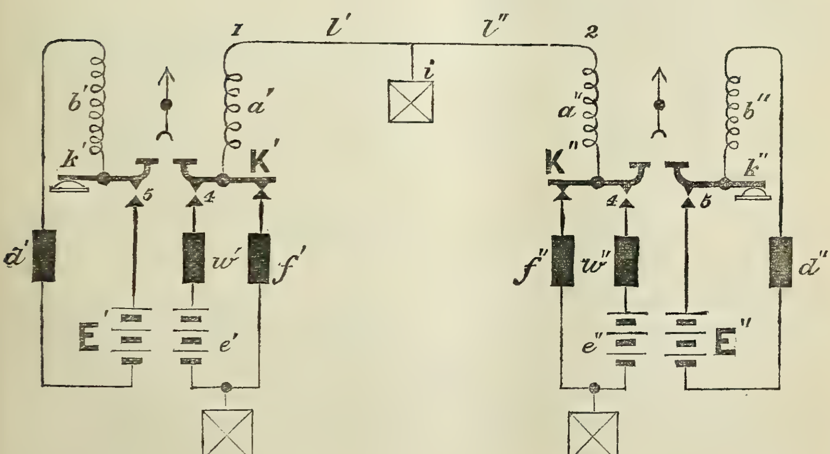
(Continued from Vol. XLIV, Part II, 1875.)

III. The compensation method.*

This method is the oldest; Fig. 3 gives the general diagram.

FIG. 3.



* Dr. Wilhelm Gintl, Director General of Telegraphs in Austria, is the inventor of this earliest method. In 1853 he made the first practical experiment on a line between Vienna and Prague (240 miles).

Explanation of diagram.

e is the E. M. F. of the line battery.

β its internal resistance.

E is the E. M. F. of the compensation battery.

α its internal resistance.

K is a constant resistance key. Dr. Gintl used an ordinary key, which, it will be obvious, must result in a failure.

k is an ordinary key; both keys, in the same station, are worked simultaneously, *i. e.*, contacts 4 and 5 are closed and broken at one and the same time.

$d, f,$ and w are certain resistances.

a is the *one* coil of the differential instrument which is connected up in the line circuit.

b is the *other* coil of the differential instrument which is connected up in the compensation circuit. By a and b shall be also designated the resistances of these two coils.

The coils a and b with their batteries e and E respectively are arranged in such a manner that they have opposite magnetic effects with respect to the same magnetic pole. The two circuits in each station (the line circuit, and the compensation circuit) are insulated from each other. All the other terms, as L, L', L'', &c., shall have the same physical meaning as before.

The compensation method has two principal defects which the two preceding methods do not possess.

Firstly. The success of working a line duplicé by the compensation method will clearly depend on the possibility of being able to close and open simultaneously two different contacts (4 and 5). The mechanical difficulty of doing so sufficiently accurately was pointed out by Dr. Werner Siemens, and in fact constitutes one of the reasons which led him to propose the differential method.

Secondly. The balance in each station may be disturbed *directly* by a variation of the electrical condition (internal resistance and E. M. F.) of the two batteries (E and e) employed.

In the preceding two methods the variation of the internal resistance of the signalling battery can only be felt *indirectly* by affecting the balance of the distant station; while the variation of E. M. F. has no effect at all. Hence a given variation in the battery or batteries must necessarily produce a greater disturbance of balance in the *compensation method* than in the two preceding ones. We know that even so-called constant galvanic batteries, doing work, alter their electrical conditions perceptibly, especially their internal resistance, and consequently this defect weighs most decidedly against the compensation method. In all other respects the compensation method has the same defects as the differential method, and in addition some others which will be understood as the investigation proceeds.

General expressions for the two functions "D" *and* "S."

To obtain the functions D, and S, we have to develop the general expressions for the forces p, P and Q, say for Station I.

$$p' = A'\, m' - B'\, n'$$

where A', and B' are the currents which pass through the two coils a' and b' respectively, when Station I is sending and Station II is at rest; m' and n' are the forces exerted by the two coils a' and b' respectively on one and the same magnetic pole, when a unit of current passes through them. At balance in Station I, $p' = o$.

Further $$P' = \mathfrak{A}'\, m'$$

where $\mathfrak{A}'$ is the current which passes through the coil a' when Station II is sending and Station I is at rest (single signals).

Further $$Q' = \forall'\, m' + \rotatebox[origin=c]{180}{B}'\, n'$$

where $\forall'$, and $\rotatebox[origin=c]{180}{B}'$ are the currents which pass through the coils a', and b' respectively when both stations are sending simultaneously (Duplex signals).

The compensation circuit, and the line circuit in each station being electrically independent of each other, we have

$$\rotatebox[origin=c]{180}{B}' = B'$$

invariably without condition.

If we further presuppose that depressing of the key K does not alter the complex resistance of the station, a condition which, for the regularity of signals, we are obliged to assume here as well as in the two preceding methods, it will be clear that

$$\forall' = \forall' + \mathfrak{A}'$$

Substituting these values for $\forall'$ and $\rotatebox[origin=c]{180}{B}'$ in the expression for Q', we get:

$$p' = A'\, m' - B'\, n'$$
$$P' = \mathfrak{A}'\, m'$$
$$Q' = (A' + \mathfrak{A}')\, m' + B'\, n'$$

The signs of the terms may be again contained in the currents, while m' and n' are taken as absolute numbers. We must only remember that $A'\, m'$, and $B'\, n'$ must be invariably of opposite sign. Arbitrarily we will call the current A positive when the negative pole of the line battery is to earth.

Now we have again two different modes of connecting up the line batteries, *viz.* :—

1*st.* The same poles of the line batteries are connected to earth in the two stations:

$$p' = \pm A' m' \mp B' n'$$
$$P' = \mp \mathfrak{A}' m'$$
$$Q' = (\pm A' \mp \mathfrak{A}') m' \mp B' n'$$

2*nd.* Opposite poles of the two line batteries are connected to earth in the two stations :

$$p' = \pm A' m' \mp B' n'$$
$$P' = \pm \mathfrak{A}' m'$$
$$Q' = (\pm A' \pm \mathfrak{A}') m' \mp B' n'$$

Subtracting in either case P' from Q', we get

$$Q' - P' = S' = p'$$

Or, on account of having fulfilled the key equation $f = w + \beta$, the difference of the forces which produce single and duplex signals is equal in sign and magnitude to the force by which balance is disturbed. Further it is, also for the compensation method, quite immaterial whether the same or opposite poles of the two line batteries are connected to earth. As pointed out, it is preferable to connect the same poles, *i. e.*, the negative poles of the line batteries to earth.

Assuming this case we have :

$$p' = A' m' - B' n'$$
$$P' = - \mathfrak{A}' m'$$
$$Q' = (A' - \mathfrak{A}') m' - B' n'$$

Substituting now for A', B', and $\mathfrak{A}'$ their values, and remembering that

$$\left.\begin{aligned} m' &= q' \sqrt{a'} \\ n' &= r' \sqrt{b'} \end{aligned}\right\} \text{approximately.}$$

we get the following general expressions for the two functions D and S:

$$\left.\begin{aligned} S' &= e' q' \frac{\Delta'}{R' K'} \\ D' &= \frac{e'}{e''} \cdot \frac{K''}{R' K'} \cdot \frac{\Delta'}{\mu'' \sqrt{a'}} \end{aligned}\right\} \text{for Station I.}$$

and

$$\left.\begin{aligned} S'' &= e'' q'' \frac{\Delta''}{R'' K''} \\ D'' &= \frac{e''}{e'} \cdot \frac{K'}{R'' K''} \cdot \frac{\Delta''}{\mu'' \sqrt{a''}} \end{aligned}\right\} \text{for Station II.}$$

where $\Delta = R \sqrt{a} - K \lambda v \sqrt{b}$

$R = a + b + d$

$$K = f + a + c$$

$$\lambda = \frac{E}{e}$$

$$v = \frac{r}{q}$$

Rigid fulfilment of the two functions S = o *and* D = o.

For finite quantities these two functions can only become zero if $\Delta = o$, *i. e.*,

$$R\sqrt{a} - K\,\lambda\,v\sqrt{b} = o.$$

which is the balance equation for the *compensation method.*

To fulfil this equation permanently, no matter what the special cause of disturbance may be, we can again adopt two essentially different modes of re-adjustment, *viz.* :—

Either leave the two coils a and b or their armatures stationary, and adjust balance by altering the resistance in one or both of the two circuits, or leave the resistances constant and alter the relative position of the two coils or their armatures with respect to a given magnetic pole. These two methods of re-adjusting balance shall be considered separately.

a. Re-adjustment of balance by altering resistances.

In order to have *immediate balance* it will be clear that the alteration of resistance must be restricted to the compensation circuit, which is electrically independent of the line circuit. The total resistance in the compensation circuit consists of three different resistances, namely b, a, and d. Neither b nor a, considering their nature, can conveniently be made adjustable in practice ; hence the alteration of resistance in the compensation circuit is restricted to d, which must therefore consist of increments of the proper size. The adjustment of d should be quick and convenient.

In addition to this adjustment, $\lambda = \frac{E}{e}$ may be made adjustable by varying E in increments of *one* cell. Such an adjustment is however not fine enough for ordinary use. The E. M. F. of one cell is too large a quantity in comparison with the total E. M. F. used in the compensation circuit. If the variation of the line current becomes very great, it might perhaps be found convenient to alter E, but as an ordinary mode of adjustment it must be dispensed with.*

* During the period of low insulation of the line it might be advisable and practicable to make E larger than during the period of high insulation of the line (wet and dry season).

It is scarcely needed to point out that to adjust balance by altering the line current, either by varying the resistance or the E. M. F.* or both of the line circuit, must be rejected once for all, because such an adjustment of balance in the one station could never take place without disturbing the balance of the other station; or in other words the required *immediate balance* could not be fulfilled.

b. Re-adjustment of balance by moving the coils or armatures.

If we suppose both the coils or their armatures simultaneously movable in the same direction, then clearly this mode of adjustment contains not only the required immediate balance, but in addition represents also a very rapid and entirely continuous action. For this reason it is apparently preferable to the first method, where the adjustment can only be carried on in *one* branch by varying d in increments.† Which of the two methods, however, is to be chosen finally, depends on other considerations which will

* Alteration of E. M. F. of a galvanic battery cannot be achieved without altering its internal resistance. Hence varying e, would also involve a variation of β, and in order to keep $f = w + \beta$, it would become necessary to alter w simultaneously with e, *i. e.*, w would have to be increased when e decreases and *vice versâ.* This method being rough, would therefore be also inconvenient.

† It has been suggested to adjust balance by a continuous variation of resistance, as for instance by moving a contact point along a thin platinum wire in the same manner as Dr. Wr. Siemens has done in his bridge employed for comparing accurately comparatively small resistances. It is, however, scarcely necessary to point out that such a method, if applied for Duplex Working, must result in a failure, at all events so long as electro-magnetic instruments are used for producing the signals. For in such a case, the resistance of any branch, no matter what special Duplex method may be employed, must bear a certain ratio to the given resistance of the line, in order to get the signals with sufficient force. This ratio, as my investigations have shewn, is by no means a small one, and hence the resistances of all branches, even for a short line, cannot be made small. Therefore the platinum wire, constituting part of one or two branches of the Duplex method employed, must also offer a considerable resistance, *i. e.*, must be of great length. Hence to alter such a large resistance continuously and perceptibly, as is indicated by the balance disturbance, must evidently involve a considerable movement of the contact point, which, even choosing the thinnest possible wire, and the shortest Telegraph line, becomes already for the daily variation so large as to make its application impossible. Unless another material of much higher specific resistance than Platinum wire can be found which, at the same time, allows of the sliding contact being made securely, the adjustment of balance by a continuous variation of resistances must be dispensed with. Such a material does not appear to exist. I thought of acting on Phillips's suggestion to use pencil-marks for the adjustable resistance, and although I found that pencil-resistances can be adjusted very accurately, and can be enclosed in a very small space, and that they keep sufficiently constant, it is difficult, if not impossible, to alter them by a sliding contact. The "*Uebergangs-widerstand*" is too variable and too great. Besides, if the contact is made with sufficient pressure, its sliding along alters the thickness of the pencil mark, and hence the resistances become inconstant and uncertain.

become clear further on. We know now that both these modes of adjustment are convenient and practicable, and contain *immediate* balance without special conditions. In fact in this respect the compensation method is preferable to the differential method where immediate balance by varying resistances could only be obtained when varying the four branches simultaneously, according to a fixed relation

Rapid approximation of the two functions S *and D towards zero.*

On account of $f = w + \beta$ we have

$$S = p = e\,q\,\frac{\Delta}{R\,K}$$

where

$$\Delta = R\sqrt{a} - K\lambda\,v\sqrt{b}$$

Now suppose $\Delta = o$, then this equation may be disturbed by K, R, λ, v, a, or b varying ; a and b are wire resistances which may be taken as constant, for their variation with temperature is exceedingly small, and in case of accident, *i. e.*, a coil breaking or becoming shunted, nothing short of actual repair could help. Further v, supposing the differential instrument to be properly designed and mechanically well executed, may be taken as a perfectly constant quantity which certainly, as long as the coils or their armatures are not moved on purpose, does not alter of its own accord.

The quantities left, which by variation may affect the balance equation, are K, R, and λ.

Of these three quantities the variation of K may become largest, for K does not only contain the line resistance, which is highly variable, but K includes also the internal resistance of both the line batteries, which, even for the best known form of galvanic battery, is by no means a constant quantity. The variation of the internal resistance of the line battery in each station produces of course the greatest disturbance of balance in *that* station.

The next quantity most liable to change of its own accord is clearly R, since it contains the internal resistance of the compensation battery.

λ, the ratio of the two E. M. F's. in one and the same station, though being also liable to change, will however vary very little. The E. M. F. of a well prepared galvanic battery, especially when the battery is worked by weak currents, is far more constant than is generally believed.*

* It appears that changes which have been observed to take place in the E. M. F. of a Minotto or Leclanche's battery are generally apparent only, not real. Such changes are generally quite within the limits of observation errors, and if they are large they are then generally due to the incorrectness of the method employed for measuring the E. M. F., or to cells actually having become exhausted. It appears that this mysterious force in each cell either exists in its full vigour, or not at all, there seems to be no continuous change in either direction.

With respect to the variation of the three quantities K, R, and λ, the function S may therefore be expressed in three different forms.

$$S_1 = e\,q\,\frac{\lambda\,v\,\sqrt{b}}{R\,K}\,\delta K \text{ when } K \text{ varies only.}$$

$$S_2 = e\,q\,\frac{\sqrt{a}}{R\,K}\,\delta R \quad \text{when } R, \textit{ i. e.}, a \text{ varies only.}$$

$$S_3 = e\,q\,\frac{v\,\sqrt{b}}{R}\,\delta\lambda \quad \text{when } \lambda, \textit{ i. e.}, E \text{ or } e \text{ or both are varying only.}$$

These three different disturbances of balance may act singly or conjointly, and it is clear that they are independent of each other, at all events as far as this investigation is concerned. Consequently the safest plan will be to make each influence as small as the circumstances will allow it.

The disturbance S_1 for any constant $eq\,\lambda\,v\,\sqrt{b}$, and any given δK will obviously become smallest the larger $R\,K$ is selected. Supposing $R + K$ constant, whatever that value finally may be, $R\,K$ has a maximum for $R = K$, and the very same condition will obviously make the disturbance S_2 smallest.

S_3 offers no best condition, this expression only shews that it has an absolute maximum with respect to b, namely as

$$R = a + d + b, \text{ for } b = a + d.$$

Thus we are informed that whatever relation between b and $a + d$ may be finally chosen, $b = a + d$ should *not* be selected, as otherwise any given variation of λ would have the greatest possible disturbing effect on the balance. But $b = a + d$ being the condition for the maximum magnetic effect in the compensation circuit, it is hereby established that for the sake of regularity of signals, which under all circumstances is to be considered of paramount importance in Duplex Telegraphy, the magnetic effect in the compensation branch *must not* be achieved in the most economical manner, but quite the reverse. This, as the compensation circuit has actually to produce wholly or partly the *duplex signals*, is a testimonium paupertatis for the compensation method, and proves it in this respect inferior to both the *double balance* and *the differential method.*

$$R = K$$

is the regularity condition for the compensation method, *i. e.*

*In order to make the disturbance of balance by a variation of the resistance in both the circuits absolutely as small as possible, the total resistance of the compensation circuit should be equal to the total resistance of the line circuit.**

* This result is against the adopted view, for Dr. Gintel as well as others after him have always treated the compensation circuit as a kind of *local circuit, i. e.*, giving to it

If we now substitute in S_1 for K the value R, and in S_2 for R the value K we get

$$S_1 = e\,q\,\frac{\lambda\, v \sqrt{b}}{R^2}\,\delta\, K$$

$$S_2 = e\,q\,\frac{\sqrt{a}}{K^2}\,\delta\, R$$

while

$$S_3 = e\,q\,\frac{v\sqrt{b}}{R}\,\delta\,\lambda$$

remains the same.

S_1 has an absolute maximum for $b = \frac{a + d}{3}$; S_2 for $a = \frac{f + c}{3}$ and S_3 for $b = a + d$ as stated before.

Hence we know what relations between the different variable, should *not* exist.

This is all we can get from the function S. For further relations we must look to the function D.

For Station I we have*

$$D' = \frac{e'}{e''}\,\frac{K''}{R'\,K'}.\,\frac{\Delta'}{\mu'\sqrt{a'}}$$

which again, with respect to the variations of K', R', and λ' may be written in three different forms:

$$D_1' = \frac{e'}{e''}\,\frac{K''}{R'\,K'}.\,\frac{\lambda'\, v' \sqrt{b'}}{\mu' \sqrt{a'}}\,\delta\, K'$$

$$D_2' = \frac{e'}{e''}.\,\frac{K''}{R'\,K'}.\,\frac{1}{\mu'}\,\delta\, R'$$

and

$$D_3' = \frac{e'}{e''}.\,\frac{K''}{R'}.\,\frac{v' \sqrt{b'}}{\mu' \sqrt{a'}}\,\delta\,\lambda'$$

---

as low a resistance as practice allows. But this is clearly wrong, for if R is made very small as compared with K, the balance becomes unstable. This fact explains, to a certain degree, the failure which has attended the application of the compensation method for Duplex working, because the method was tried under the most unfavorable quantitative arrangements.

* When investigating the minimum absolute magnitude of S, the terms could be taken without an accent, because S contains only terms belonging to the same station. When investigating D this cannot be done as D contains also terms belonging to the other station.

Considering that

$$\frac{K''}{K'} = \frac{i + l'' + \rho''}{i + l' + \rho'}$$

$$\mu' = \frac{i}{i + l' + \rho'}$$

and

$$\frac{K''}{\mu'} = L + \rho' + \rho'' + \frac{(l' + \rho')\,(l'' + \rho'')}{i}$$

we have

$$D_1' = \frac{e'}{e''} \cdot \frac{i + l'' + \rho''}{i} \qquad \frac{\lambda'\, v' \sqrt{b'}}{R' \sqrt{a'}} \cdot \delta K'$$

$$D_2' = \frac{e'}{e''} \cdot \frac{i + l'' + \rho''}{i} \cdot \frac{1}{R'} \delta R'$$

$$D_3' = \frac{e'}{e''} \left\{ L + \rho' + \rho'' + \frac{(l' + \rho')\,(l'' + \rho'')}{i} \right\} \frac{v' \sqrt{b'}}{R' \sqrt{a'}} \delta \lambda'$$

put $\qquad \dfrac{e'}{e''} = s$

$$\frac{i + l'' + \rho''}{i} = J$$

$$L + \rho' + \rho'' + \frac{(l' + \rho')\,(l'' + \rho'')}{i} = T$$

and $\dfrac{1}{R' \sqrt{\dfrac{a'}{b'}}} = \dfrac{1}{\psi'}$

$$\therefore \quad D_1' = sJ\,\lambda'\, v' . \frac{1}{\psi'}\, \delta K'$$

$$D_2' = sJ \frac{1}{R'} \delta R'$$

$$D_3' = sv' \frac{T}{\psi'}\, \delta\lambda'$$

Now keeping s, J, λ', and v' constant, D_1' becomes smallest for any given $\delta K'$ the larger ψ' is selected ; while D_3' becomes smallest for any given $\delta\,\lambda'$ the smaller $\dfrac{T}{\psi'}$ is selected, and D_2' becomes smallest the larger R' is chosen.

Now $\psi' = R' \sqrt{\dfrac{a'}{b'}}$ has a maximum for $a' = b'$;

for $R' = b' + a' + d' = b' + \gamma'$, and putting $\gamma' = b'\,t'$ we have

$\psi' = (1 + t') \sqrt{a'\,b'}$ which, for $a' + b'$, and t' constant, has clearly a maximum for $a' = b'$. This proceeding is right, because we take b' as the original variable, and vary a' and γ' simultaneously with b', in order to keep t' and $a' + b'$ constant; while J and s are independent of a', b', and γ'.

In order to be sure that $a' = b'$ makes also D_3' a minimum, we must shew that T keeps constant, *i e.*, ρ' keeps constant when a' varies. But $\rho' = a' + f'$, thus we have only to consider f' simultaneously variable with a' equal and opposite to the variation of a, which is allowed. Therefore the condition $a' = b'$ makes undoubtedly the disturbances D_1' and D_3' minima. While the disturbance D_2', which contains R' in the denominator only, is not affected by this relation, but depends on the absolute value of b' only, which should be chosen as large as possible.

$a = b$ is therefore the *second* regularity condition, the fulfilment of which makes the relative disturbance of balance by a variation of K and λ as small as possible.

Substituting now $a' = b'$ in the expression of the D disturbances and remembering that

$$R' = K'$$

we get

$$D_1' = s\,\lambda'\,v'\,\frac{J}{K'}\,\delta K'$$

$$D_2' = s\,\frac{J}{K'}\,\delta R'$$

$$D_3' = s\,v'\,\frac{T}{K'}\,\delta\lambda'$$

Thus D_1', and D_2', for constant s, λ', and v', become smallest the smaller $\frac{J}{K'}$ is, while D_3' becomes smallest the smaller $\frac{T}{K'}$ is.

Now remembering that

$$J = \frac{i + l'' + \rho''}{i}$$

$$K' = \frac{(l'' + \rho'')\,(i + l' + \rho') + i\,(l' + \rho')}{i + l'' + \rho''}$$

and

$$T = L + \rho' + \rho'' + \frac{(l' + \rho')\,(l'' + \rho'')}{i}$$

$$\therefore\ \frac{J}{K'} = \frac{(i + l'' + \rho'')^2}{i\,\{(l'' + \rho'')\,(i + l' + \rho') + i\,(l' + \rho')\}}$$

$$\frac{T}{K'} = \frac{i + l'' + \rho''}{i} = \mu$$

For a tolerably good line $l'' + \rho''$ as well as $l' + \rho'$ can be taken as small in comparison with i; hence approximately

$$\frac{J}{K'} = \frac{1}{l' + l'' + \rho' + \rho''} = \frac{1}{L + \rho' + \rho''} \text{ and}$$

$$\frac{T}{K'} = 1$$

From which it follows that also for the compensation method ρ' and ρ'' should be selected as large as possible.

But $\rho = a + f$ does not give a condition besides that we know we should select a and f absolutely *not* small.

Further we see that the disturbance D_3' has v' for its factor, while D_1' has $\lambda' v'$ for its factor.

Hence for a given $\lambda' v'$, the best will he to make v' as small as possible.

The regularity of the signals is therefore obtained if we fulfil the following conditions in either station.

$$R = K$$
$$a = b$$
$$\rho \quad \text{as large as possible}$$
$$v \quad \text{as small as possible.}$$

Knowing this we may now consider that balance in either station is rigidly obtained, or that

$$R \sqrt{a} - K \lambda v \sqrt{b} = o$$

but $\qquad R = K$

and $\qquad a = b$

we have $\qquad \lambda v = 1$

The absolute value of a may now be determined by considering that it is advisable to produce the signals in either station in the most economical manner.

Maximum Magnetic Moment.

We have

$$P' = \frac{e''}{a'' + f'' + c''} \mu' q' \sqrt{a'}$$

$$P'' = \frac{e'}{a' + f' + c'} \mu'' q'' \sqrt{a''}$$

But

$$\frac{\mu'}{a'' + f'' + c''} = \frac{\mu''}{a' + f' + c} = \frac{i}{Q}$$

where $\qquad Q = i\,(L + \rho' + \rho'') + (l' + \rho')\,(l'' + \rho'')$

$$\therefore \quad P = P' + P'' = i\,\frac{e'' q' \sqrt{a'} + e' q'' \sqrt{a''}}{Q}$$

which has a maximum for a' and a'' taken as independent variables.

If we, for instances, take $i = \infty$, than

$$P = \frac{e\,q\,\sqrt{a}}{L + 2\,(a + f)}$$

$$\therefore \quad a = \frac{L}{2} + f \text{ for a perfect line, and by inference}$$

$$\left.\begin{array}{l} a = \dfrac{L'}{2} + f' \\ a'' = \dfrac{L''}{2} + f'' \end{array}\right\} \text{ approximately.}$$

Now we can decide on the method to be adopted for re-adjusting balance. On account of the regularity condition $R = K$, and as both undergo variation, especially K, we are obliged to adjust balance in the compensation branch by varying the resistance d, and leave the coils or their armatures stationary.

Thus the general solution of the 1st problem for the compensation method is:

1. Readjustment of balance is to be effected by a variation of resistance in the compensation circuit and *not* by a movement of the coils or their armatures. By this adjustment R is kept equal to K permanently, no matter in which branch the variation takes place.

2. $\mathbf{f = w + \beta}$

$$\mathbf{a = b = \frac{L}{2} + f}$$

$$\mathbf{v\lambda = 1}$$

v as small as possible and λ as large as possible.

β is known from the number and nature of the single cells of which the battery has to consist to produce through the given line (connected up in a circuit like Fig. 3) single signals with sufficient strength.

w is known from the absolute largest variation β may undergo in time; hence f is determined and therefore also a and b.

Determination of λ and v.

We know that $\lambda\, v = 1$, and further that $\lambda = \dfrac{E}{e}$ should be selected as large as possible or v as small as possible, but otherwise it appears that no fixed values for λ and v can be ascertained. If, however, we consider the nature of the variations of R and K, which may disturb the balance, *viz.*: those variations of R and K which are due to unavoidable decrease of the internal resistance of the two batteries by the working currents, it will be seen that a best value of λ does exist, and that therefore v also becomes fixed.

Suppose that at a certain moment

$R = K$ is rigidly fulfilled, and remembering that

$R = b + d + a$

$K = 2\ (a + f) + L$ (for a perfect line, *i. e.*, $i = \infty$)

and further, that

$$a = b$$

and $f = w + \beta$

we have $d + a = a + 2\ w + 2\ \beta + L.$

Now, in this equation suppose everything constant except a and β, the internal resistance of the two batteries E and e respectively. Hence if we could achieve that

$$\delta\ a = 2\ \delta\beta \text{ invariably,}$$

the variation of the internal resistance of the two batteries would not disturb the equation $R = K$, and therefore also not affect the balance. With absolute certainty we cannot fulfil this desirable relation between the two variations, but with some probability we may. For it is well known that the internal resistance of a galvanic battery decreases in time by the current passing through the battery. Hence, if we suppose that the two batteries consist of identical cells (equal in nature, size, and internal resistance) we may say that the variation of the internal resistance of a single cell by the unit current in the unit of time is the same for both the batteries. Further, if we make the other not improbable supposition, that the variation at any one time is proportional to the current which passes at that time, we have

$$\delta\ a = \epsilon\ E.\ \frac{E}{R + \delta\ R}\ \phi^{(t)} = \epsilon \frac{E^2}{R}\ \phi^{(t)}$$

and

$$2\ \delta\ \beta = \epsilon\ e.\ \frac{e}{K + \delta\ K}\ \phi^{(t)} = \epsilon \frac{e^2}{K}\ \phi^{(t)}$$

where ϵ is the variation of the internal resistance of a single cell in unit of time by unit of current; $\phi^{(t)}$ a certain unknown function of the time which, as the two batteries are working simultaneously, is not required to be known.

Hence from $\delta\ a = 2\ \delta\ \beta$

and $K = R$

it follows that $\lambda = \frac{\mathrm{E}}{\mathrm{e}} = \sqrt{2}$

and $\mathrm{v} = \frac{\mathrm{r}}{\mathrm{q}} = \sqrt{\frac{1}{2}}$

These values of λ and v bring the compensation method, with respect to regularity of working, as close to the differential method as is possible for us to do. For the disturbance of balance in the sending station by the steady decrease of the internal resistance of the two batteries has now been *probably* eliminated, which defect is excluded from the other two methods, by their own nature. There are then remaining only those variations of the battery resistance which do not follow the law of steady decrease, but which are more accidental, and make therefore the compensation method still inferior to either the differential or bridge method.

Physical meaning of $v \sqrt{\frac{1}{2}}$

It has been proved that balance in each station is to be established by adjusting resistance and *not* by a movement of the coils or their armatures. Hence it will be practical and convenient to coil the two helices above each other, and have them acting on one and the same iron core.

Further as $v = \frac{r}{q} = \sqrt{\frac{1}{2}}$ it follows that the magnetic action of the a coil must be made greater than that of the b coil. Therefore it will be best to coil the helix b on the top of the helix a.

Further the magnetic action of a cylindrical coil of resistance a (in Siemens units) can be expressed as follows:

$$m = s \sqrt{a} \sqrt{\frac{A \lambda}{c\, l}}$$

where A is half the cross section of the coil (cut by a plane through the axis of the coil) expressed in $\square^{\text{m m.}}$

λ the absolute conductivity of the wire material ($H_g = 1$ at $O°$ $C.$)

l the length of an average convolution expressed in metres.

s the magnetic force exerted by an average convolution of the coil when the unit of current passes.

c a coefficient representing the manner of coiling.

Hence for the a coil we have

$$m_a = s' \sqrt{a} \sqrt{\frac{A' \lambda'}{c' l'}} = q \sqrt{a}$$

for the b coil

$$m_b = s'' \sqrt{b} \sqrt{\frac{A'' \lambda''}{c'' l''}} = r \sqrt{b}$$

Dividing m_b by m_a, and remembering that by condition $a = b$, and that $\lambda' = \lambda''$, $c' = c''$ by necessity, we have:

$$v = \frac{r}{q} = \frac{s''}{s'} \sqrt{\frac{A'' l'}{A' l''}}$$

As we have supposed that the magnetic action of any one cylindrical coil is proportional to the magnetic action* of an average convolution it is also consistent to put $s' = s''$, and we have at last

$$\frac{A'' \, l''}{A' \, l'} = \frac{1}{2}$$

If now the two bobbins of the coils *a* and *b* are taken of equal length, and if the thickness of the *a* coil be d', the thickness of the *b* coil d'', and the diameter of the iron core $2\,r$, we have,

$$\frac{A''}{A'} = \frac{d''}{d'}.$$

$$l' = (2\,r + d')\,\pi$$

$$l'' = \{2\,(r + d') + d''\}\,\pi$$

$$\therefore \qquad (4\,r + d')\,d'' = 2\,d'\,(r + d')$$

This equation fixes the relative dimensions of the two bobbins and their cores in order to have $v = \sqrt{\frac{1}{2}}$

Suppose for instance we make $d' = d''$ arbitrarily† we get $2\,r = d$, and from it can be easily calculated that the diameter of the wire of the *b* coil should be *about* 19 *per cent. larger* than that of the *a* coil. The absolute diameter of the wire depends of course on the absolute dimensions of the bobbins, and on the resistance of the line for which the instrument is to be used. But this question, although of practical importance, has nothing to do with the Theory of Duplex Telegraphy. This settles the solution of the 1st problem of the compensation method.

* Lenz and Jacobi have experimentally proved that, within certain limits, the magnetic force exerted by a convolution on its centre (iron core) is almost independent of the diameter of the convolution. These limits are generally fulfilled in Telegraph Construction. Hence the magnetic action of a coil can be put proportional to the magnetic action of *one* convolution. Theoretically this can of course not be true, for the magnetic force exerted by a convolution necessarily extends on both sides of the plane in which the convolution is situated. Therefore the wider a convolution is the less of its total force exerted will be made use of for producing magnetism in the iron core, and, consequently, the force exerted by a convolution on its centre must decrease with the diameter of the convolution. It appears, however, that this decrease is exceedingly slow, and in the present investigation it is considered unnecessary to be taken into account.

† I have not been able to find anywhere a definite law which connects the diameter of a coil with the diameter of the core acted upon. In Siemens' relay, an instrument so well considered in all its details of construction, the diameter of the coil is about three times the diameter of the core. In the absence of anything else on the subject I thought myself justified in using this proportion. Hence the substitution of $d' = d''$, which gives $d = 2\,r$, or total diameter of the *a* coil equal to three times the diameter of the iron core.

Other methods. There have been suggested, from time to time, many other methods of duplex working. On a closer examination it will, however, be found that, as a general rule, they do not differ essentially from the three fundamental methods treated of. I shall therefore dispense with the labour of investigating these derived methods.

In case it should be thought necessary to investigate them, no difficulties ought to be met with, if only the general plan of attacking duplex problems be remembered, *viz.*, to draw the diagram of the method in its most general form; develop the forces p, P, and Q; from these three forces determine the functions S and D; find the relations which must hold between the different variables (resistances and E. M. F. S.) of which the system consists, in order to make S and D simultaneous minima; consider the question of *immediate balance* which determines also the best mode of adjusting balance; consider that the movement of the key must not alter the complex resistance of the station to which the key belongs, *i. e.*, that the working of the key must not affect the balance of the distant station; determine the absolute values of the different variables when balance is rigidly fulfilled by considering the question of economy, *i. e.*, establish the relations for maxima currents and maxima magnetic moments; any variables which should then be left indeterminate must be fixed by secondary considerations, and by certain practical conditions.

Before comparing quantitatively the efficiency of the three fundamental methods treated of, it is requred to solve two questions, *viz.*:—The E. M. F. required for each duplex method; the absolute size of the increments of the adjustable resistance.

[To be continued.]